On free convection melting of a solid immersed in a hot dissimilar fluid

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Abstract—The flowfield, heat transfer and the resulting melting rate of a solid body immersed in an otherwise quiescent, hot fluid are considered. The problem is an interesting example of free convection, differing from the usual free convection problems in that the dominant buoyancy force is due to the fluid–melt density difference rather than the familiar thermal expansion. An asymptotic solution is obtained in analytical form for the limiting case of negligible thermal buoyancy effect, negligible mixing between the melt and the ambient fluid, large ambient fluid Prandtl number and small Stephan number $c_p \Delta T/\lambda_c$. The neglected parameters are shown to be small for common fluid combinations. The theory is also applicable to miscible fluids with large Lewis numbers. A qualitative and semi-quantitative comparison with experimental results is given.

1. INTRODUCTION

THE MELTING of a solid body immersed in an otherwise quiescent, hot fluid different from its own molten or sublimed phase constitutes an interesting sub-class of the phenomenon of free convection. After a short transient, the melting rate is mediated by the quasi-steady, natural convective motion of the melt and the ambient fluid. As a rule, the density difference between the melt and the ambient fluid is greater than that due to thermal expansion. Consequently, the buoyancy force responsible for the motion of the melt layer derives primarily from differences in material densities rather than from thermal gradients in the fluid.

Many examples of this problem can be found. The melting of ice in saline water and in alcoholic drinks and the melting of wax and butter in hot water come to mind. The sublimation of dry ice in air is another example. It can be expected that the problem may also arise in metallurgical and chemical processing. Recent interest, however, has been spurred by considerations of the integrity of structures when immersed in molten core material after a hypothetical reactor accident. For example, one problem which has received considerable attention is the heat transfer from a pool of heat generating molten core debris (see review by Glueckler and Baker [1]). Experimental studies on the rate of enlargement of an internally heated, molten pool were conducted by Farhadieh and Baker [2]. The result does not appear to correlate with free convection heat transfer rates based on thermal buoyancy considerations for non-melting pools. This indicates that the density difference between the pool and the melt layer might have played a dominant role in controlling the heat transfer process.

A question which comes to mind immediately is whether a distinction needs to be made between miscible and immiscible fluid combinations. There are two facets to this question. First, for immiscible fluids, surface tension would be present. However, for sufficiently large bodies, surface tension will play only a small role in the motion of the fluids. Secondly, for miscible fluids, the composition distribution in the boundary layer must be considered. It can be expected that for large Lewis number ($\equiv Sc/Pr$), the mass diffusivity is much lower than the heat diffusivity. Relatively abrupt composition variations will be found within the thermal boundary layer. This would be the case for most liquid combinations and some gas combinations. For these materials, the thin diffusional boundary layers should not significantly affect the nature of the problem.

The present communication is concerned with the basic prototype of such a melting problem. The formulation is carried out for a nearly vertical melting surface for a solid whose molten phase is not miscible with the surrounding fluid. The results should be approximately applicable to miscible fluids with large Schmidt numbers. A closed form solution is provided for the limiting case of large Prandtl numbers and small Stephan numbers.

2. GOVERNING EQUATIONS

We adopt the usual practice in boundary-layer analysis in designating x as the coordinate along the solid–fluid interface and y the coordinate normal to the interface (see Fig. 1). Since the solid boundary is gradually receding, the coordinate system is time dependent. However, in the limit of slow melting rate, the process can be considered quasi-steady. The governing equations for incompressible fluids are:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + g(\rho_{a\infty} - \rho)\cos\gamma \quad (1)$$

NOMENCLATURE

c	specific heat	ζ	abbreviation for $f''(\eta_i)$, dimensionless velocity of the ambient fluid–melt interface
f	similarity variable for streamfunction ψ		
k	thermal conductivity	ρ	density
S	modified Stephan number, see equation (24)	$\Delta\rho$	density difference between the melt and the ambient fluid
T	temperature	τ	abbreviation for η_i (an indication of melt layer thickness)
u, v	velocity components along x and y, respectively		
V	melting rate (in velocity unit)	ψ	streamfunction.
x, y	coordinates (see Fig. 1).		

Greek symbols

Subscripts
a   ambient fluid
c   see λ_c

α	heat diffusivity	i	interface between the ambient fluid and the melt
γ	angle of inclination (see Fig. 1)		
ζ	abbreviation for $\theta(\eta_i)$, dimensionless temperature of the ambient fluid–melt interface	m	melt layer
		ms	melt–solid interface (same as sm)
η	similarity variable for y, see equation (17)	s	solid
		sm	solid–melt interface (same as ms)
θ	dimensionless temperature	∞	at $y = \infty$.
λ	latent heat of fusion		
λ_c	effective latent heat including sensible heat contribution, see equation (14)	Superscripts	
		*	stretched variables based on external viscous boundary-layer scaling
μ	viscosity		
ν	kinematic viscosity	+	stretched variables based on external thermal boundary-layer scaling.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\rho c u \frac{\partial T}{\partial x} + \rho c v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right). \tag{3}$$

These equations govern both the melt layer and the boundary layer in the hot ambient fluid. The bound-

FIG. 1. Schematic representation of an immersed solid with an exposed, nearly vertical surface, showing the coordinate system employed.

ary conditions are:

$$T \to T_{a\infty}, \quad u \to 0 \quad \text{as } y \to \infty \tag{4}$$

$$T = T_{ms}, \quad u = 0, \quad v = \frac{\rho_s}{\rho_m} V(x) \text{ at } \quad y = 0 \tag{5}$$

where T_{ms} is the melting point and $V(x)$, the melting rate, is an unknown. The coupling between $V(x)$ and the temperature distribution will be discussed later. It is convenient to solve equations (1)–(3) separately for the two fluid layers assuming constant properties for each layer and matching the two solutions at the interface. The matching conditions for the tangential velocity, u, normal velocity, v, and temperature, T, are:

$$\left.\begin{aligned} u_m &= u_a \\ \rho_m v_m &= \rho_a v_a = 0 \\ \mu_m \frac{\partial u_m}{\partial y} &= \mu_a \frac{\partial u_a}{\partial y} \end{aligned}\right\} \quad y = \delta_m \tag{6a, b, c}$$

$$\left.\begin{aligned} T_m &= T_a \\ k_m \frac{\partial T_m}{\partial y} &= k \frac{\partial T_a}{\partial y} \end{aligned}\right\} \quad y = \delta_m \tag{7a, b}$$

where the subscript m denotes the melt layer and the subscript a denotes the thermal boundary layer of the ambient fluid. Note that an additional condition $v = 0$ is incorporated with equation (6b). The problem is not overspecified since δ_m is unknown.
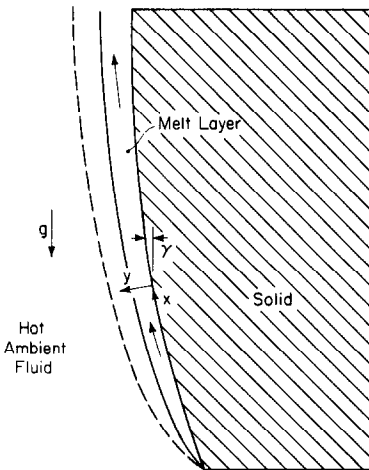
The coupling between the velocity and heat transfer boundary conditions at the melting boundary is provided by consideration of heat conduction in the solid. Consistent with the boundary-layer assumption employed for the fluid layers, we may neglect the axial conduction in the solid. Thus the energy equation for the solid is:

$$\rho_s c_s V \frac{\partial T_s}{\partial y} = k \frac{\partial^2 T}{\partial y^2} \qquad (8)$$

with boundary conditions

$$T_s = T_{sm} \quad \text{at } y = 0 \qquad (9)$$

$$T_s \to T_{s\infty} \quad \text{as } y \to -\infty. \qquad (10)$$

The solution is

$$(T_s - T_{s\infty}) = (T_{sm} - T_{s\infty}) e^{Vy/\alpha_s} \qquad (11)$$

where α denotes the heat diffusivity.

From the jump condition for the temperature gradient at the solid–melt interface

$$k_m \frac{\partial T_m}{\partial y} = \rho_s V \lambda + k_s \frac{\partial T_s}{\partial y} \quad \text{at } y = 0 \qquad (12)$$

it follows that

$$k_m \frac{\partial T_m}{\partial y} = \rho_s V \lambda_c = \rho_m v_m \lambda_c \quad \text{at } y = 0 \qquad (13)$$

where λ_c is an abbreviation, representing an effective latent heat defined as

$$\lambda_c \equiv \lambda + c_s (T_{sm} - T_{s\infty}). \qquad (14)$$

Equation (13) together with equation (5) serves as the coupling between the velocity and heat transfer boundary conditions.

3. SIMILARITY TRANSFORMATION

We shall assume that

$$\gamma \ll 1. \qquad (15)$$

Accordingly,

$$\cos \gamma = 1 + O(\gamma^2). \qquad (16)$$

Thus, although x is measured along the surface of the melting solid, which may not necessarily form a vertical surface, it is possible to replace $\cos \gamma$ by unity with only second-order error. The governing equations will thus admit a similarity solution just as a vertical surface.

We introduce the similarity variable

$$\eta = \left(\frac{g \lambda_c \rho_m \Delta \rho}{k_m \mu_m \Delta T} \right)^{1/4} x^{-1/4} y \qquad (17)$$

and the dimensionless temperature and streamfunction θ and f

$$\theta(\eta) \equiv \frac{T - T_{sm}}{\Delta T} \qquad (18)$$

$$f(\eta) \equiv \left(\frac{\lambda_c^3 \rho_m^3 \mu_m}{k_m^3 \Delta T^3 g \Delta \rho} \right)^{1/4} x^{-3/4} \psi(x, t) \qquad (19)$$

where ψ is the streamfunction and ΔT denotes $T_{a\infty} - T_{sm}$, the temperature difference between the undisturbed ambient fluid and the solid melting point.

The momentum and the energy equations are then reduced to the ordinary differential equations

For the melt layer:

$$f_m''' + 1 = \frac{1}{Pr_m} S \left[\frac{1}{2} (f_m')^2 - \frac{3}{4} f_m f_m'' \right] \qquad (20)$$

$$\theta_m'' = -\tfrac{3}{4} S f_m \theta_m' \qquad (21)$$

For the boundary layer in the ambient fluid:

$$f_a''' = \frac{v_m}{v_a} \frac{1}{Pr_m} S \left[\frac{1}{2} (f_a')^2 - \frac{3}{4} f_a f_a'' \right] \qquad (22)$$

$$\theta_a'' = -\frac{3}{4} \frac{\alpha_m}{\alpha_a} S f_a \theta_a' \qquad (23)$$

where primes denote differentiation with η. Pr_m and Pr_a are the Prandtl number for the two materials. S denotes the modified Stephan number defined as

$$S \equiv c_m (T_{a\infty} - T_{sm}) / \lambda_c. \qquad (24)$$

The boundary conditions are

$$\theta_m(0) = 0, \quad f_m(0) = -(3/4)\theta_m'(0), \quad f_m'(0) = 0,$$
$$\theta_a(\infty) = 1, \quad f_a'(\infty) = 0. \qquad (25a-e)$$

The matching conditions at the interface $\eta = \eta_i$ are

$$\theta_a(\eta_i) = \theta_m(\eta_i) \quad [\equiv \theta(\eta_i)] \qquad (26a)$$

$$k_a \theta_a'(\eta) = k_m \theta_m'(\eta_i) \qquad (26b)$$

$$f_a(\eta_i) = f_m(\eta_i) = 0 \quad [\equiv f(\eta_i)] \qquad (26c)$$

$$f_a'(\eta_i) = f_m'(\eta_i) \qquad (26d)$$

$$\mu_a f_a(\eta_i) = \mu_m f_m(\eta_i). \qquad (26e)$$

Note that the extra condition contained in equation (26c) is to be employed to determine the unknown interface position, η_i.

Two observations can be made from an examination of equations (20)–(25):

1. The equations for the melt layer can be immediately decoupled from those of the external boundary layer if the interfacial temperature $\theta(\eta_i)$ and the interfacial tangential velocity $f'(\eta_i)$ are known. Once decoupled, these equations can be solved by conventional procedures.

2. Although the equations are not overly difficult, there are a large number of parameters—four in the differential equations and two in the boundary conditions. Therefore, while exact solutions are clearly possible for these coupled ordinary differential equations, the results are expected to be at best clumsy and at worst confusing.

Accordingly, we shall proceed to examine some limiting conditions for which the differential equations can be significantly simplified. Our procedure will be: (1) to obtain solutions of the decoupled differential equations in terms of three quantities η_i, $\theta(\eta_i)$ and $f'(\eta_i)$; (2) to express η_i and $f'(\eta_i)$ in terms of $\theta(\eta_i)$; and (3) to solve for $\theta(\eta_i)$ algebraically. For convenience, these three parameters will be denoted as

$$\eta_i \equiv \tau \text{ (reduced melt layer thickness)} \quad (27a)$$

$$\theta(\eta_i) \equiv \zeta \text{ (a–m interface temperature)} \quad (27b)$$

$$f'(\eta_i) \equiv \xi \text{ (a–m interface velocity).} \quad (27c)$$

Limiting form as $\mathrm{Pr}_a \to \infty$

Note that our definition of the dimensionless parameters are based on melt layer scaling. The structure of the boundary layer in the ambient fluid can be clarified if the momentum equation, equation (22), is transformed to stretched variables according to its own scaling. The new variables are

$$\eta^* = (\eta - \tau)\left[\xi \frac{v_m}{v_a}\frac{S}{Pr_m}\right]^{1/2} \quad (28a)$$

and

$$f^* = f\left[\frac{1}{\xi}\frac{v_m}{v_a}\frac{S}{Pr_m}\right]^{1/2}. \quad (28b)$$

We may call this scaling the external viscous scale. The transformed equation is

$$f^{*'''} = \tfrac{1}{2}f^{*'1/2} - \tfrac{3}{4}f^*f^{*''} \quad (29)$$

with boundary conditions

$$f^*(0) = 0, \quad f^{*'}(0) = 1 \quad \text{and} \quad f^{*'}(\infty) = 0 \quad (30a\text{–}c)$$

where primes denote differentiation with respect to the stretched variable η^*. Note that equations (29) and (30) uniquely define a value $f^{*''}(0)$ which is of the order unity. The shear stress matching condition for the interface thus becomes

$$f_m''(\eta_i) = \frac{\mu_a}{\mu_m} f_a''(\eta_i)$$

$$= \frac{\mu_a}{\mu_m} \xi^{3/2}\left(\frac{v_m}{v_a}\frac{S}{Pr_m}\right)^{1/2} f^{*''}(0). \quad (31)$$

Equation (31) serves as one of the boundary conditions of the melt layer momentum equations. With $f^{*''}(0) = O(1)$, it is clear that

$$f_m''(\tau) \to 0 \quad \text{as} \quad \xi^{3/2}\left(\frac{\rho_a\mu_a}{\rho_m\mu_m}\frac{S}{Pr_m}\right)^{1/2} \to 0. \quad (32)$$

We may also rescale the external energy equation using stretched variables based on the following external thermal scale

$$\eta^+ = \left(\xi\frac{\alpha_m}{\alpha_a}S\right)^{1/2}(\eta - \tau) \quad (33a)$$

$$f^+ = \left(\frac{1}{\xi}\frac{\alpha_m}{\alpha_a}S\right)^{1/2} f \quad (33b)$$

$$\theta^+(\eta^+) = \frac{\theta(\eta) - \zeta}{1 - \zeta} \quad (33c)$$

the transformed equation is

$$\theta^{+''} = -\tfrac{3}{4}f^+\theta^{+'} \quad (34)$$

with boundary conditions

$$\theta^+(\infty) = 1, \quad \theta^+(0) = 0. \quad (35a,b)$$

By comparing equation (28a) with (33a), the ratio of the thermal scale over the viscous scale can be found to be $Pr_a^{-1/2}$. In other words, for large Pr_a, the viscous scale length and viscous boundary-layer thickness greatly exceed the thermal scale length and thermal boundary thickness. Thus, the velocity $f'(\eta)$ can be considered essentially uniform within the thermal boundary layer. From equation (27c)

$$f'(\eta) \to \xi, \quad f^+(\eta^+) \to 1, \quad f(\eta) \to \xi\eta, \quad f^+ \to \eta^+$$

$$\text{for} \quad 0 < \eta^+ < O(1) \quad \text{as } Pr_a \to \infty. \quad (36a\text{–}d)$$

The equation for the thermal boundary layer thus becomes

$$\theta^{+''} = -\tfrac{3}{4}\eta^+\theta^{+'} \quad \text{for} \quad Pr_a = \infty. \quad (37)$$

It is seen that the assumption of infinite Prandtl number leads to two significant simplifications: (1) a simpler boundary condition for the melt layer; and (2) a simpler differential equation for the external thermal boundary layer.

It is interesting to recall that for single-phase thermal boundary layers, whether forced or free convection, the infinite Prandtl number assumption incurs very little error for most liquids and only about a 20% error even for gases with $Pr \simeq 0.7$. Thus the simpler equations are expected to retain the essential physics of the problem and be quantitatively meaningful as well.

The limit $S \to 0$

We note that equation (20) and (21) can be significantly simplified if S/Pr_m and S are small:

$$f_m''' + 1 = 0 \quad \text{for} \quad S/Pr_m = 0 \quad (38)$$

$$\theta_m'' = 0 \quad \text{for} \quad S = 0. \quad (39)$$

These two assumptions are related to the neglect of inertia and specific heat, respectively, for the melt layer. Similar assumptions were adopted by Nusselt [3] in his original analysis of film-wise condensation. As shown by perturbation analysis [4] and numerical film-layer computations [5], the parameters S and S/Pr are usually quite small and their neglect does not significantly alter either the qualitative or quantitative aspects of the condensation problem. The magnitudes of these parameters are similarly small in the present problem—typically less than 0.1—and their neglect is hence justified as a first approximation.

4. SOLUTION

The solution of equation (39) subject to the boundary conditions, (25a) and (27b), is

$$\theta_m = (\zeta/\tau)\eta. \qquad (40)$$

Equation (25b) can now be written as

$$f_m(0) = -\frac{4}{3}\frac{\zeta}{\tau}. \qquad (41)$$

With equations (25c), (32) and (41) as boundary conditions, the solution of (38) is

$$f_m = \frac{1}{2}\tau\eta^2 - \frac{1}{6}\eta^3 - \frac{4}{3}\frac{\zeta}{\tau}. \qquad (42)$$

The dimensionless melt-layer thickness τ can be solved in terms of the fractional thermal resistance ζ from equation (26c):

$$\tau = (4\zeta)^{1/4}. \qquad (43)$$

This now permits the evaluation of interfacial tangential velocity $\xi = f'(\tau)$ from equation (42) and (43) as

$$\xi = \zeta^{1/2}. \qquad (44)$$

Note that the solution for equation (37) with boundary conditions, equation (35), can be obtained by straightforward procedures to yield

$$\theta^+(\eta^+) = \text{erf}\,\frac{3^{1/2}}{8^{1/2}}\eta^+ \qquad (45)$$

whence

$$\theta^{+\prime}(0) = (3/2\pi)^{1/2}. \qquad (46)$$

Making use of equations (33a, c) and (44), the temperature gradient at the interface is found to be

$$\theta'_a(\tau) = (3/2\pi)^{1/2}(1-\zeta)[\zeta^{1/2}(\alpha_m/\alpha_a)S]^{1/2}. \qquad (47)$$

The melt side temperature gradient can be evaluated from equations (40) and (43) as

$$\theta'_m(\tau) = \frac{1}{4^{1/4}}\zeta^{3/4}. \qquad (48)$$

The jump condition, equation (26b), thus leads to an equation for ζ

$$\zeta/(1-\zeta)^2 = A \qquad (49)$$

where

$$A = \frac{3k_a\rho_a c_a}{\pi k_m\rho_m c_m}S. \qquad (50)$$

Equation (49) is in a convenient form when A is small. If A is large, a more convenient form can be obtained by solving equation (49) algebraically:

$$\zeta = 1 + \frac{1}{2A} - \left(\frac{1}{4A^2} + \frac{1}{A}\right)^{1/2}. \qquad (51)$$

Making use of equations (40) and (41), expressions

for the melting rate or the heat transfer coefficient based on the temperature difference $\Delta T = T_{a\infty} - T_{ms}$ can be obtained. Specially, the local Nusselt number at a distance x from the leading edge is

$$Nu_x \equiv \frac{xh}{k_m} \equiv \frac{xg}{k_m(T_{a\infty} - T_{ms})}$$

$$= 1/(2^{1/2})\left(\frac{1}{S}\frac{\Delta\rho}{\rho_m}\frac{gx^3}{\nu_m\alpha_m}\right)^{1/4}\zeta^{3/4}. \qquad (52)$$

The mean Nusselt number for a surface of total length L is

$$\overline{Nu_L} = 2(2^{1/2})/3\left(\frac{1}{S}\frac{\Delta\rho}{\rho_m}\frac{gL^3}{\nu_m\alpha_m}\right)^{1/4}\zeta^{3/4}. \qquad (53)$$

The melting rate V is related to the heat transfer rate q by the relationship

$$V = \frac{q}{\rho_s\lambda}. \qquad (54)$$

5. EXPERIMENTAL COMPARISONS

As a semi-quantitative check of the basic premises of the above theory for immiscible liquids, the melting rate of a vertical paraffin cylinder immersed in a hot aqueous solution of potassium iodide was measured. The cylinder had an initial diameter of 94.4 mm and a temperature of 24°C. The temperature of the solution was 62°C. The melting point of the paraffin was about 54°C. The specific gravities of the solution and the molten paraffin were 1.087 and 0.810, respectively. After a 19.5-min immersion, the cylinder was removed and the final dimensions measured with a caliper. The observed melting rate vs height is shown in Fig. 2. The data are compared with the theoretical melting rate predicted from equations (52)–(55), shown as the solid line. It is seen that there is good qualitative and semi-quantitative agreement. The melting rate decreased with height, indicating that the melt layer and the boundary layer flowed upward due to the upward
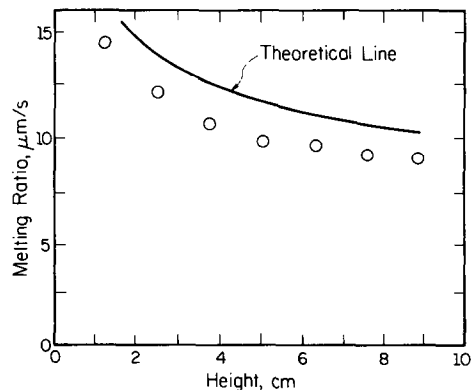

FIG. 2. Melting of paraffin cylinder in a potassium iodide solution $\rho_a/\rho_m = 1.34$.

buoyancy force of the molten wax. The thermal buoy-ancy force of the ambient thermal boundary layer is directed downward. This could have accounted for the fact that the experimental melting rates were lower than those predicted from theory which neglected thermal buoyancy effects. It should also be noted that the physical properties of paraffin are strongly tem-perature dependent and not known with precision. A more quantitative comparison with the essentially zeroth-order theory is probably not justified.

It is interesting to note that for the conditions employed, the predicted value of ζ was about 0.12. In other words, the dominant thermal resistance was the thermal boundary layer in the ambient fluid. In this light, the semi-quantitative agreement was quite remarkable because the theory attributed all of the buoyancy force to the melt layer which constituted only 12% of the heat transfer resistance.

The melting rates of a material whose molten phase is soluble in the ambient fluid have been reported by Farhadieh and Zimmerman [6]. A rectangular block of polyethylene glycol, a water-soluble wax, was insu-lated on five sides, with one remaining vertical surface exposed to an aqueous solution of potassium iodide maintained by a thermostat-controlled heater at a temperature of 65°C. The initial temperature of the polyethylene glycol was 21°C and its melting point was 45°C. The melting of the solid was photo-graphically recorded and the melting rate was obtained by analyzing the photographs. The density of the potassium iodide solution could be altered by

changing the salinity. The observed mean melting rate as a function of the density difference is shown in Fig. 3 compared to the theory. This rate represented the average for the entire vertical surface. Good quali-tative and semi-quantitative agreements were obtained. Again, in view of the imprecisely known properties of the materials involved, a more quan-titative comparison with the theory is not justified. The fact that experimental values appeared higher than the theoretical values could be due to the rela-tively strong convection currents generated by the heating wires immersed in the solution.

6. CONCLUDING REMARKS

The experiments cited above appear to corroborate the basic postulates of the present theory: (a) the dominant driving force in an immersion melting prob-lem is the buoyancy due to the melt–ambient fluid density difference instead of thermal buoyancy; and (b) the miscibility of the two fluids plays no major role in the basic processes. The theory, of course, was only a zeroth-order asymptotic theory. Theories account-ing for the higher order terms and more refined experi-ments are clearly desirable.

FIG. 3. Average melting rate of polyethylene glycol in hot aqueous solution of potassium iodide

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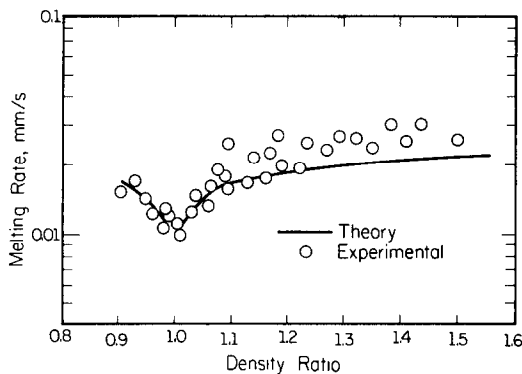
SUR LA CONVECTION NATURELLE ASSOCIEE A LA FUSION D'UN SOLIDE IMMERGE DANS UN FLUIDE CHAUD

Résumé—On considère le champ d'écoulement, le transfert thermique et la fusion associée d'un solide immergé dans un autre fluide chaud au repos. Le problème est un exemple intéressant de convection naturelle, différent des problèmes habituels en ceci que la force d'Archimède dominante est due à la différence de densité du fluide de fusion plutôt qu'à la dilatation thermique. Une solution asymptotique est obtenue sous forme analytique pour le cas limite d'un effet d'Archimède thermique négligeable, d'un mélange négligeable entre le bain fondu et le fluide ambiant, d'un nombre de Prandtl ambiant élevé et d'un petit nombre de Stephan $c_p \Delta T/\lambda_c$. Les paramètres négligés sont montrés être faibles pour les combinaisons de fluides miscibles avec de grands nombres de Lewis. On donne une comparaison qualitative et semi-quantitative avec des résultats expérimentaux.

ÜBER DIE KONVEKTION BEIM SCHMELZEN EINES FESTSTOFFS IN EINEM HEISSEN ANDERSARTIGEN FLUID

Zusammenfassung—Strömungsbild, Wärmeübergang und die daraus resultierende Schmelzrate eines festen Körpers, der in ein sonst ruhendes, heißes Fluid getaucht ist, wurden untersucht. Dies ist ein interessantes Beispiel für freie Konvektion, das sich von sonstigen Fällen dadurch unterscheidet, daß der Auftrieb nicht durch thermische Dichteunterschiede hervorgerufen wird, sondern durch den Dichteunterschied zwischen Fluid und Schmelze. Es wird eine asymptotische Näherungslösung in analytischer Form angegeben für den Grenzfall verschwindenden thermischen Auftriebs, vernachlässigbarer Vermischung von Schmelze und Fluid, großer Prandtl- und kleiner Stephan-Zahl $c_p \cdot \Delta T/\lambda_c$ des Fluids. Es wird gezeigt, daß für gebräuchliche Fluidkombinationen die vernachlässigten Parameter klein sind. Die Theorie läßt sich auch auf mischbare Fluide mit großer Lewis-Zahl anwenden. Ein qualitativer und teilweise quantitativer Vergleich mit Meßergebnissen wird durchgeführt.

О СВОБОДНОКОНВЕКТИВНОМ ПЛАВЛЕНИИ ТВЕРДОГО ТЕЛА, ПОМЕЩЕННОГО В ГОРЯЧУЮ ЖИДКОСТЬ СО СВОЙСТВАМИ, ОТЛИЧАЮЩИМИСЯ ОТ СВОЙСТВ ТВЕРДОГО ТЕЛА

Аннотация—Рассматривается поле течения, теплоперенос и результирующая скорость плавления твердого тела, погруженного в неподвижную горячую жидкость со свойствами, отличающимися от свойств тела. Задача является интересным примером свободной конвекции, не похожим на обычные задачи по свободной конвекции, в которых преобладающая подъемная сила вызвана разностью плотностей жидкости и расплава, а не тепловым расширением. Асимптотическое решение получено в аналитической форме для предельного случая пренебрежимо малых величин подъемной силы за счет теплового эффекта, смешения между расплавом и окружающей жидкостью, большого числа Прандтля окружающей жидкости и малого числа Стефана $c_p \Delta T/\lambda_c$. Показано, что пренебрегаемые параметры малы для типичных ситуаций. Теория также применима к смешивающимся жидкостям с большими числами Льюиса. Дано качественное и количественное сравнение с экспериментальными результатами.